

# Hanbury Brown–Twiss interferometry with electrons: Coulomb vs. quantum statistics\*

Gordon Baym<sup>a</sup> and Kan Shen<sup>a,b</sup>

<sup>a</sup>*Department of Physics, University of Illinois, 1110 W. Green Street, Urbana, IL 61801*

<sup>b</sup>*Quantitative Strategies, Credit Suisse, 11 Madison Ave, New York, NY 10010C*

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A longstanding goal of Akira Tonomura was to observe Hanbury Brown–Twiss anti-correlations between electrons in a field-emission free electron beam. The experimental results were reported in his 2011 paper with Tetsuji Kodama and Nobuyuki Osakabe [1]. An open issue in such a measurement is whether the observed anti-correlations arise from quantum statistics, or are simply produced by Coulomb repulsion between electrons. In this paper we describe a simple classical model of Coulomb effects to estimate their effects in electron beam interferometry experiments, and conclude that the experiment did indeed observe quantum correlations in the electron arrival times.

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## I. INTRODUCTION

Since the pioneering detection by Hanbury Brown and Twiss (HBT) of “bunching” of photons in a light beam [2], HBT experiments with massive bosons, e.g., atomic beams [3, 4] and  $\pi$  and  $K$  mesons in high energy nuclear collisions [5, 6] have shown similar two-particle correlations. Seeing anti-correlation – or anti-bunching – effects in experiments with identical fermions where the two-particle intensity ( $I$ ) correlation function

$$C(\mathbf{r}_2 - \mathbf{r}_1, t_2 - t_1) = \frac{\langle I_1 I_2 \rangle}{\langle I_1 \rangle \langle I_2 \rangle}. \quad (1)$$

should fall, at small separations (either in position or momentum space), to zero for particles of the same spin (or to 1/2 for unpolarized particles) has proven more elusive. Experiments with neutrons are complicated by a low energy nuclear resonance, while experiments with protons are complicated in addition by Coulomb repulsion [7, 8]. On the other hand, anti-bunching of neutral cold fermionic  $^{40}\text{K}$  atoms [9], and the corresponding bunching of neutral cold bosonic  $^{87}\text{Rb}$  atoms [10] emitted from optical lattices has been successfully observed. In addition, anti-bunching with neutral atomic  $^3\text{He}$  beams, as well as bunching with neutral atomic  $^4\text{He}$  atomic beams, was clearly demonstrated in the experiment of Jelte et al. [4].

Detecting anti-bunching in a beam of electrons has been a major experimental challenge over the years, owing to the low degeneracy as well as the short coherence time of the beams. Starting in the 1990’s Akira Tonomura and his group focussed on seeing this striking effect of quantum statistics with electrons in a field-emission electron beam. Following his group’s theoretical feasibility analysis [11, 12], electron HBT experiments have

been realized in free space [1, 13]; such experiments with electrons show a reduction in the correlation function for small space-time separation, generally attributed to anti-bunching. On the other hand, repulsive Coulomb interactions between electrons also reduce the probability of two electrons being close in space. Whether the observed anti-bunching effect is due to electron quantum statistics or rather Coulomb repulsion is the issue we deal with in this paper. We conclude that the recently reported experiment of Kodama, Osakabe, and Tonomura [1] very cleanly sees HBT in the arrival time correlations of electrons pairs; In the experiment of Ref. 14, at a significantly lower beam energy, Coulomb effects account for several percent of the HBT signal.

HBT interferometry has in fact been seen for electrons in semiconductor devices, [14, 15] where, owing to screening, Coulomb effects are less important. For example, in the HBT experiment of Ref. 16, the screening length ( $\sim 5\text{nm}$ ) is typically much smaller than the Fermi wavelength ( $\sim 40\text{nm}$ ) [16]. Interestingly, two dimensional mesoscopic semiconductors open the possibility of seeing HBT correlations for fractional statistics [17, 18] as well as Aharonov-Bohm physics [19].

The importance of correcting for Coulomb interactions has long been recognized in interpreting high energy nuclear experiments [20]. For example, the raw correlation function of distinguishable pions of opposite charge ( $\pi^+\pi^-$ ), produced in the E877 ultra-relativistic heavy ion collision experiment [21], shows a very similar buildup at small momentum differences to those of identical charged pions ( $\pi^+\pi^+$  and  $\pi^-\pi^-$ ). The Coulomb interaction between opposite charges tends to increase the probability of a pair of bosons being close in momentum, while reducing that for like charges. Only after the effects of Coulomb interactions are extracted, does one see the expected effects of quantum statistics (see, e.g., Refs. 7 and 22).

Our aim in this note is to present a simple schematic discussion of Coulomb effects in interferometry experiments with electrons, based on the classical behavior of electrons taken pairwise. We do not attempt to explain

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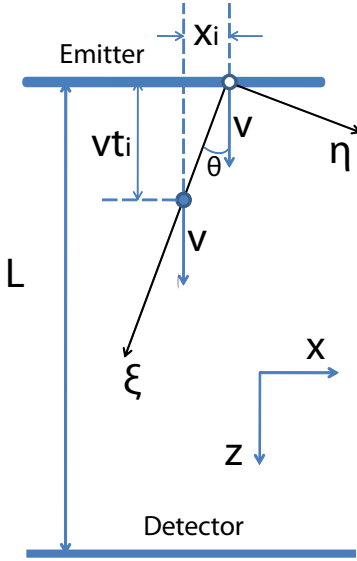


FIG. 1: Schematic of emission of two electrons. The first (closed circle) and second (open circle) travel downward to the detector plate at velocity  $\simeq v$ . Coulomb acts along the  $\xi$  direction.

the detailed results of Tonomura's group on HBT with electrons, but rather aim to estimate the role of Coulomb interactions in their search for quantum correlations.

The Coulomb problem for a pair of electrons is characterized by four length scales, 1) the electron Bohr radius  $a_0 \equiv \hbar^2/mc^2$ , with  $m$  the electron mass; 2) the size  $r_0$  of the emitting region transverse to the beam; 3) the typical separation  $z_0$  of the particles along the beam direction; and importantly, 4) the classical turning point of the pair,  $r_{tp}$ , defined by  $e^2/r_{tp} = q^2/2m_{red}$ , where  $q$  is magnitude of the final relative momentum of the two particles and  $m_{red} = m/2$ .

The traditional method of correcting for Coulomb interactions is to employ the Gamow correction, which assumes that the characteristic separation of the pair of particles is much smaller than their classical turning point, namely, that the particles are produced well within the classically forbidden regime bounded by  $r_{tp}$ . The actual rate observed in an experiment is taken to be that in the absence of Coulomb interactions times the Gamow correction,  $|\psi_c(0)|^2$ , which is the absolute value squared of the relative Coulomb wave function at the origin,

$$\psi_c(0) = \left( \frac{2\pi\eta}{e^{2\pi\eta} - 1} \right)^{1/2}, \quad (2)$$

where the dimensionless parameter  $\eta$  equals  $zz'e^2/\hbar v_{rel}$ , with  $v_{rel}$  the relative velocity of the two particles of charges  $ze$  and  $z'e$ . On the other hand, in a field emission source, the relative energy of a pair is  $\simeq \Delta E$ , where  $\Delta E$  is the initial electron energy spread in the beam. Thus  $r_{tp} \simeq e^2/\Delta E \simeq (1.46/\Delta E_{eV})$  nm, where  $\Delta E_{eV}$  is the energy spread measured in electron volts. Since  $r_{tp}$  is

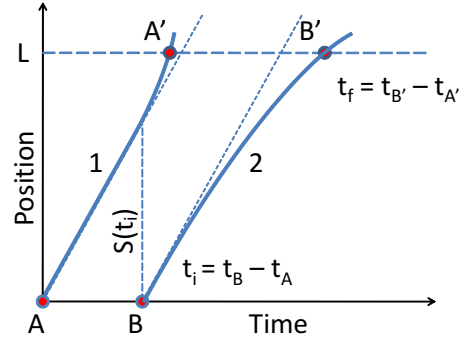


FIG. 2: Schematic picture of the effect of Coulomb repulsion on the trajectories of two electrons emitted successively from the same point in the tip.

generally tens of nm,  $r_{tp} \ll r_0$ , indicating that a pair of electrons in such an experiment is typically emitted *outside* the pair's classical turning point. Coulomb effects are dominated by classical physics rather than by a quantum Gamow correction.

## II. CLASSICAL MODEL

To bring out the effect of Coulomb interactions, we model the experiment as independent emissions of electrons from a tip, followed by acceleration to final velocity  $v$  in the beam direction ( $z$ ) and energy  $E_f = mv^2/2$ . We first neglect quantum statistics, and focus on the Coulomb effects in a single pair of particles, since the major contribution to the correlation function is from particles nearby in space and time, a configuration in which we can, to a first approximation, neglect many-body effects. We assume that the emission points of the pair are separated by  $x_i$  in space and  $t_i$  in time. Thus the initial spatial separation of the pair is  $s_i = (x_i^2 + (vt_i)^2)^{1/2}$ .

The Coulomb repulsion between the electrons increases their relative separation in space and time, as is illustrated schematically in Fig. 2 for two electrons emitted at the same point in the tip at times  $t_A$  and then  $t_B$ . The electron separation at later time is readily calculated from the conservation of energy of the relative motion of the two electrons,  $E_{rel} = m\dot{s}^2/4 + e^2/s$ . A lower bound on the size of the Coulomb hole can be derived by neglecting the initial relative kinetic energy of the pair; then after integration of  $\dot{s}$ , one finds that the final separation of the pair,  $s_f$ , is determined implicitly by

$$\sqrt{\frac{4e^2}{m}} \Delta t = s_i^{3/2} \left\{ \sqrt{\sigma(\sigma - 1)} + \ln [\sqrt{\sigma} + \sqrt{\sigma - 1}] \right\}, \quad (3)$$

where  $\sigma = s_f/s_i$ , and  $\Delta t \simeq L/v$  is the time elapsed be-

tween emission and detection of the pair. Approximately,

$$s_f = \begin{cases} s_c (s_c/s_i)^{1/2}, & s_i \leq s_c \\ s_i, & s_i > s_c, \end{cases} \quad (4)$$

where  $s_c \equiv v\tau_c \equiv (2e^2L^2/E_f)^{1/3}$  defines a critical Coulomb distance and time  $\tau_c$ ; numerically,  $s_c \simeq 6.5 \times 10^{-4} L_{cm}^{2/3} / E_{f,KeV}^{1/3}$  cm.

As seen in the left panel of Fig. 3, the relation between the final separation  $s_f$  when the electrons reach the detector plate and  $s_i$  is not monotonic, but rather is decreasing at small  $s_i$  and increasing at large  $s_i$ . For very small initial separation, the Coulomb interaction significantly accelerates the two electrons away from each other, making the final separation large, while for very large initial separation, the Coulomb interaction is negligible, and the final separation is essentially equal to that initially. No matter how small  $s_i$  is, the final spatial separation is finite, i.e., there is a *Coulomb hole* in the distribution of final separations. Coulomb forces increase the spatial separation between a pair of electrons,  $s(t)$ , with time, so that  $s_f > s_i$ .

Since the angle  $\theta$  that the relative position vector of the electrons makes with respect to the beam axis (see Fig. 1) is conserved in the motion, the final separations at detection are related by  $x_f/x_i = t_f/t_i = s_f/s_i$ ; thus the final separation in time of the two electrons is given by  $t_f = (s_f/s_i)t_i$ . At small  $t_i/\tau_c$ , the final  $t_f$  has the structure shown in the right of Fig. 3, dependent on  $x_i$ . However, for  $t_i/\tau_c \gtrsim 0.1$ , the curves on the right of Fig. 3 converge to that in the left panel of Fig. 3, which has a minimum at  $s_f \approx s_c$ . To a first approximation, the minimum  $t_f \approx s_c/v = \tau_c$  is independent of the initial spatial separation.

Experimentally one measures the correlation function  $C(t_f)$  in terms of the subsequent time intervals  $t_f$  between detection of particles, averaged over the distribution of initial emission intervals  $t_i$ . For independent emissions, the distribution of times between adjacent emissions from the tip is Poissonian

$$P_0(t_i) = \frac{1}{\bar{t}_i} e^{-t_i/\bar{t}_i}, \quad (5)$$

where  $\bar{t}_i$  is the average time separation between two emissions. In the absence of Coulomb corrections and quantum statistics,  $C_0(t_i) = 1$ . The final  $C(t_f)$  and  $P(t_f)$  are given in terms of the map (3) between  $s_f$  and  $s_i$ ; since the map is not simply one-to-one, one needs to sum over the two branches. The resulting  $P(t_f)$  and  $C(t_f)$ , calculated with the approximate solution (4) are shown as thin lines in Fig. 4.

In experiments, the finite time resolution of the detectors would smooth out the sharp Coulomb holes in Fig. 4. Measurement of an observable  $\tilde{f}(t)$  at time  $t$  averages the actual  $f(t')$  over  $t'$ , weighted by the detector time resolution function  $R(t - t')$ :

$$\tilde{f}(t) = \int_{-\infty}^{+\infty} R(t - t') f(t') dt', \quad (6)$$

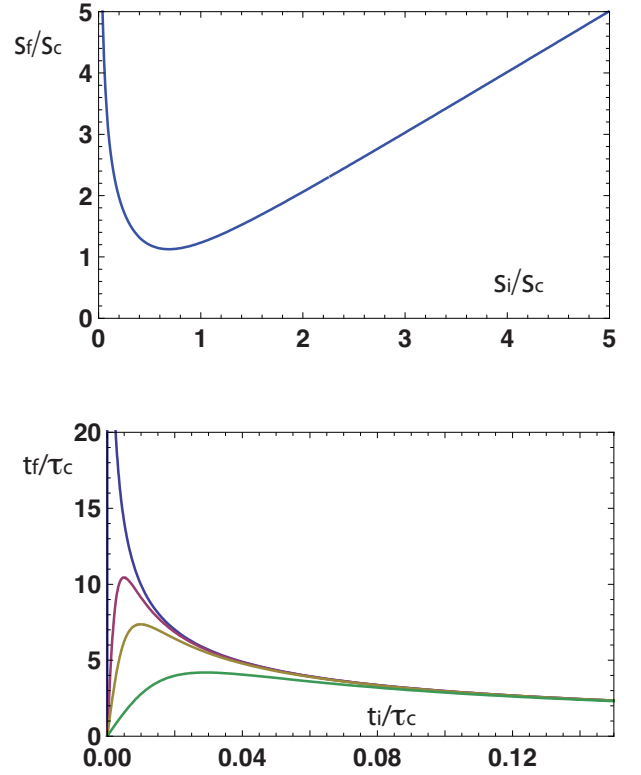


FIG. 3: a) Final vs. initial spatial separations, with Coulomb interactions included classically, and b) final vs. initial time separations for different initial transverse separations  $x_i = 0, 5, 10$ , and  $30$  nm (top to bottom). For large  $t_i/\tau_c$ , the curves all converge to that in the left panel.

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with  $f(t) = f(-t)$  for  $t < 0$ . In Fig. 4, effects of time resolution are indicated by the thick curves, where we take a Gaussian time resolution function

$$R(t - t') = \frac{1}{\sqrt{2\pi}t_r} e^{-(t-t')^2/2t_r^2}, \quad (7)$$

with a characteristic time scale  $t_r$  chosen here for illustration to equal  $\tau_c$ . Typically,  $t_r \gg \tau_c$ . The dip at low  $t_f$  in right panel illustrates clearly how Coulomb correlations can mimic quantum correlations.

### III. HBT ANTI-CORRELATIONS

To assess the importance of the Coulomb hole it is necessary to compare it with the regime of suppression of the correlation function from quantum statistics. The Pauli principle suppresses the correlation function between same spin electrons emitted from nearby points on a time scale

$$t_{HBT} = \hbar/T_f, \quad (8)$$

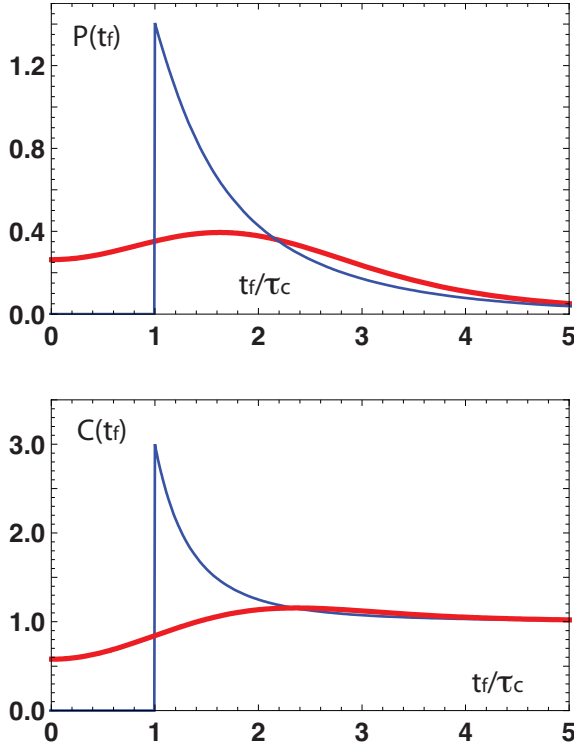


FIG. 4: Effects of Coulomb repulsion as seen in a) the final time distribution  $P(t_f)$  vs.  $t_f$ , with  $\bar{t}_i = 0.2$  ns; the vertical axis is in units of  $1/\bar{t}_i$ , and b) the normalized second-order final correlation function  $C(t_f)$  vs.  $t_f$ . The thin lines are calculated directly from Eq. (4); the thick lines include finite time resolution via Eqs. (6) and (7), with  $t_r = \tau_c$ .

where  $T_f$  is the effective longitudinal temperature of the electron gas after acceleration. Since the acceleration of the beam is essentially adiabatic, the entropy per electron is conserved, and the temperature of the gas falls with expansion. Owing to acceleration the density  $n$  of the gas drops, since in a steady state the current  $nv$  remains constant from emission through acceleration. To estimate the final gas temperature, we note that the entropy per particle of a gas with an anisotropic temperature depends on  $T^{1/2}T_\perp/n$ , where  $T$  is the longitudinal and  $T_\perp$  the transverse temperature. Thus in anisotropic expansion,  $T^{1/2}T_\perp/n$  remains constant, and for  $T_\perp$  constant,  $Tv^2$  is invariant. Initially  $v = \sqrt{T_i/m}$  and after acceleration to velocity much greater than that of the thermal motion,  $v = \sqrt{2E_f/m}$ . Thus the final longitudinal temperature of the beam is given by  $T_f \sqrt{2E_f/m} \simeq T_i \sqrt{T_i/m}$ , so that with Eq. (8) and  $T_i \simeq 2\Delta E$ , we have  $T_f = 2(\Delta E)^2/E_f$  (essentially the result derived heuristically in Ref. 1) and

$$t_{HBT} = \frac{\hbar E_f}{2(\Delta E)^2}. \quad (9)$$

The ratio of the Coulomb to HBT suppressions is thus

$$\frac{t_{HBT}}{\tau_c} \simeq 2^{-5/6} \left(\frac{a_0}{L}\right)^{2/3} \frac{E_f^{11/6} \text{Ry}^{1/6}}{(\Delta E)^2} = 0.9 \frac{E_{f,\text{KeV}}^{11/6}}{\Delta E_{\text{eV}}^2 L_{\text{cm}}^{2/3}}. \quad (10)$$

This simple calculation indicates the importance, in an HBT measurement of correlations in arrival times, of accelerating the electrons to a large final beam energy in order to overcome Coulomb effects.

In addition to time anti-correlations in the beam, HBT correlations should appear in the electron spatial separations, analogous to the spatial correlations seen in the original Hanbury Brown–Twiss measurement of the angular diameter of the star Sirius [2]. When two electrons are emitted at the same time, but spatially separated, the correlation function is suppressed in space on a scale of order the particle wavelength divided by the angular size of the source, or

$$s_{HBT} \simeq \frac{\hbar L}{mvr_0} = 6 \times 10^{-4} \frac{L_{\text{cm}}}{E_{f,\text{KeV}}^{1/2} r_{10\text{nm}}} \text{ cm} \quad (11)$$

where  $r_{10\text{nm}}$  is the transverse size of the emission region in units of 10 nm. Comparing with the minimal Coulomb hole,  $s_c$ , we find

$$\frac{s_{HBT}}{s_c} \simeq \frac{L_{\text{cm}}^{1/3}}{E_{f,\text{KeV}}^{1/6} r_{10\text{nm}}}. \quad (12)$$

To reach this ratio, the initial spatial separation must be of order  $s_c$ ; however, a more realistic bound on  $s_i$  is the transverse size of the emission tip, which leads to a considerably larger Coulomb hole, as one can infer from Fig. 1.

In the experiment of Kodama, Osakabe, and Tonomura [1]  $E_f \sim 50$ –100 KeV, and  $\Delta E \simeq 0.17$  eV; with  $L \sim 100$  cm one estimates that  $t_{HBT}/\tau_c \sim (2-6) \times 10^3$ , sufficiently high that one does not need to worry about Coulomb effects in measuring pure time-of-arrival correlations. On the other hand, in the opposite regime, when measuring spatial HBT correlations, one has  $s_{HBT}/s_c \sim 0.5$ , indicating that Coulomb effects must be taken into account in analyzing the experiment.

In contrast, in the lower energy experiment of Kiesel et al. [13], where  $E_f \sim 0.9$  KeV,  $\Delta E \simeq 0.13$  eV, and  $L \sim 1$  cm, one has  $t_{HBT}/\tau_c \sim 44$ , and thus Coulomb effects while small are not entirely negligible. For spatial correlations, however,  $s_{HBT}/s_c \sim 0.25$ , and thus Coulomb effects are dominant.

In conclusion, a full analysis of the HBT experiment of Kodama, Osakabe, and Tonomura requires correcting for Coulomb effects. The simple model presented here forms a useful and simple basis for including Coulomb interactions among the electrons. A detailed analysis requires taking into account the distribution of initial spatial separations and velocities of the electrons in addition to the

time separations, the effects of realistic finite time resolution, as well as the effects of the quadrupole magnets which give one the freedom to adjust the angular size of the beam. While Coulomb effects are present independent of the relative spin of the electron pair, Pauli quantum correlations occur only between same spin electrons; thus to distinguish optimally Coulomb repulsions from quantum correlations one would ideally like to repeat the experiments with spin polarized electron beams.

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